PHYS 301
Electricity and Magnetism

Dr. Gregory W. Clark

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- More with Dirac delta function
- Integral vector calculus

Today!

THE DIRAC DELTA FUNCTION

In 3D:

$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)$$

where $\vec{r} \equiv \hat{i}x + \hat{j}y + \hat{k}z$

and $\displaystyle\int_{all\ space} \delta^3(\vec{r})d\tau = 1$

$$\therefore \int_{all\ space} f(\vec{r})\delta^3(\vec{r}-\vec{r}_o)d\tau = f(\vec{r}_o)$$

So ....

it turns out that

$$\vec{\nabla}\cdot\left(\frac{\vec{\rho}}{\rho^3}\right) = 4\pi\delta^3(\vec{\rho})$$

$$\forall\ \vec{\rho} \equiv \vec{r} - \vec{r}_o$$

including

$\vec{r}_o = 0$

ELECTROSTATICS

OR, FOR CONTINOUS CHARGE DISTRIBUTIONS,

$$\vec{E}(x,y,z) = \frac{1}{4\pi\varepsilon_o}\int_{Vol} \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\,\rho(\vec{r}')\,d\tau'$$

- NOTE: $\rho(r')$ is completely general!

E.g., consider

$\rho(r') = \sigma\,\delta(r' - R)$

$\rho(r') = \lambda\,\delta(x' - a)\,\delta(y')$

$\rho(r') = \rho_o$

$\rho(r') = \delta(x' - a)\,\delta(y' - b)\,\delta(z' - c)$

Gauss' Law:

Integral Form:

$$\oint_{surf} \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\varepsilon_o}$$

Differential Form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_o}$$

Note:

$$q = \int_{vol} \rho(\vec{r}\,')d\tau$$

Integral Calculus: Curls

- The **fundamental theorem of curls**:

$$\boxed{\int_{\text{surface}} \left(\vec{\nabla} \times \vec{V}\right) \cdot d\vec{A} = \oint_{\text{line}} \vec{V} \cdot d\vec{l}}$$

- Also called ***STOKES' THEOREM***

- NOTE: $\left\{ \begin{array}{l} \oint \left(\vec{\nabla} \times \vec{V}\right) \cdot d\vec{A} = 0 \ \forall \text{ closed surfaces} \\ \int \left(\vec{\nabla} \times \vec{V}\right) \cdot d\vec{A} \text{ depends only on the boundary;} \end{array} \right.$

not on the surface!

e.g., soap bubble!