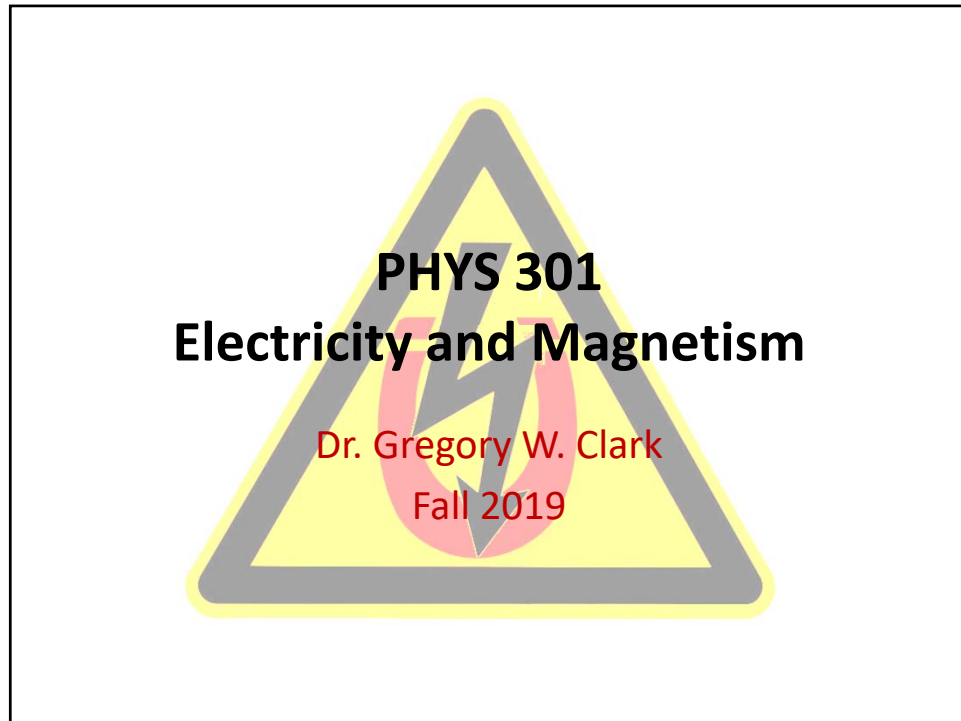




- More with Dirac delta function
- Integral vector calculus

Today!

THE DIRAC DELTA FUNCTION

In 3D:

$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)$$

$$\text{where } \vec{r} \equiv \hat{i}x + \hat{j}y + \hat{k}z$$

$$\text{and } \int_{\text{all space}} \delta^3(\vec{r}) d\tau = 1$$

$$\therefore \int_{\text{all space}} f(\vec{r}) \delta^3(\vec{r} - \vec{r}_o) d\tau = f(\vec{r}_o)$$

So

it turns out that

$$\vec{\nabla} \cdot \left(\frac{\vec{\rho}}{\rho^3} \right) = 4\pi \delta^3(\vec{\rho})$$

$$\forall \vec{\rho} \equiv \vec{r} - \vec{r}_o$$

including

$$\vec{r}_o = 0$$

ELECTROSTATICS

OR, FOR CONTINUOUS CHARGE DISTRIBUTIONS,

$$\vec{E}(x, y, z) = \frac{1}{4\pi\epsilon_o} \int_{\text{Vol}} \frac{\hat{r}}{r^2} \rho(\vec{r}') d\tau'$$

- NOTE: $\rho(r')$ is completely general!

E.g., consider

$$\rho(r') = \sigma \delta(r' - R)$$

$$\rho(r') = \lambda \delta(x' - a) \delta(y')$$

$$\rho(r') = \rho_o$$

$$\rho(r') = \delta(x' - a) \delta(y' - b) \delta(z' - c)$$

GAUSS' LAW:

INTEGRAL FORM: $\oint_{surf} \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\epsilon_o}$

DIFFERENTIAL FORM: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$

NOTE:

$$q = \int_{vol} \rho(\vec{r}') d\tau$$

Integral Calculus: Curls

- The **fundamental theorem of curls**:

$$\int_{\text{surface}} (\vec{\nabla} \times \vec{V}) \cdot d\vec{A} = \oint_{\text{line}} \vec{V} \cdot d\vec{l}$$

- Also called **STOKES' THEOREM**

- NOTE: $\left\{ \begin{array}{l} \oint (\vec{\nabla} \times \vec{V}) \cdot d\vec{A} = 0 \quad \forall \text{ closed surfaces} \\ \int (\vec{\nabla} \times \vec{V}) \cdot d\vec{A} \text{ depends only on the boundary;} \\ \text{e.g., soap bubble!} \quad \text{not on the surface!} \end{array} \right.$